TRANSMITTAL LETTER TO THE UNITED STATES DESIGNATED/ELECTED OFFICE (DO/EO/US) CONCERNING A FILING UNDER 35 U.S.C. § 371	Attorney Docket No. F40.12-0005
	U.S. Application No. **10/089026**

INTERNATIONAL APPLICATION PCT/FR 00/02716	INTERNATIONAL FILING DATE 29.09.2000	PRIORITY DATE CLAIMED 29.09.1999

TITLE OF INVENTION
METHOD FOR TRANSMITTING AN OFFSET MODULATED (BFDM/OM) MULTICARRIER SIGNAL

APPLICANT(S) FOR DO/EO/US
SIOHAN Pierre et al.

Applicant herewith submits to the United States Designated/Elected Office (DO/EO/US) the following items and other information:

1. [X] This is a **FIRST** submission of items concerning a filing under 35 U.S.C. 371.

2. [] This is a **SECOND** or **SUBSEQUENT** submission of items concerning a filing under 35 U.S.C. 371.

3. [X] This is an express request to begin national examination procedures (35 U.S.C. 371(f). The submission must include items (5), (6), (9) and (20) indicated below.

4. [X] The US has been elected by the expiration of the 19th month from the priority date (Article 31).

5. [X] A copy of the International Application as filed (35 U.S.C. 371(c)(2))
    a. [X] is transmitted herewith (required only if not transmitted by the International Bureau).
    b. [X] has been communicated by the International Bureau.
    c. [] is not required, as the application was filed in the United States Receiving Office (RO/US).

6. [X] A translation of the International Application into English (35 U.S.C. 371(c)(2)).
    a. [X] is attached hereto.
    b. [] has been previously submitted under 35 U.S.C. 154(d)(4).
    c. [] is not required, as the application was filed in English

7. [X] Amendments to the claims of the International Application under PCT Article 19 (35 U.S.C. 371(c)(3))
    a. [] are attached hereto (required only if not transmitted by the International Bureau).
    b. [] have been transmitted by the International Bureau.
    c. [] have not been made; however, the time limit for making such amendments has NOT expired.
    d. [X] have not been made and will not be made.

8. [] A translation of the amendment to the claims under PCT Article 19 (35 U.S.C. 372(c)(3)).

9. [X] An unexecuted oath or declaration of the inventor(s) (35 U.S.C. 371(c)(4)).

10. [X A translation of the annexes to the International Preliminary Examination Report under PCT Article 36 (35 U.S.C. 37(c)(5)).

Items 11. to 17. Below concern document(s) or information included:
11. [X] An Information Disclosure Statement under 37 CFR 1.97 and .198.

12. [] An assignment document for recording. A separate cover sheet in compliance with 37 C.F.R. 3.28 and 3.31 is included.

13. [X] A **FIRST** preliminary amendment.

14. [] A **SECOND** or **SUBSEQUENT** preliminary amendment.

15. [] A substitute specification.

16. [] A change of power of attorney and/or address letter.

17. [] A second copy of the published international application under 35 U.S.C. 154(d)(4).

18. [] A second copy of the English language translation of the international application under 35 U.S.C. 154(d)(4).

19. [] Other items or information: a. [X] _Eight (8)_ sheets of drawings.
   b. [X] Abstract typed on a separate page.
    c. [X] File data sheet.

U.S. APPLICATION NO. 10/089026	INTERNATIONAL APPLICATION NO. PCT/FR00/02716	ATTORNEY'S DOCKET NUMBER F40.12-0005

20. [X]   The following fees are submitted:
BASIC NATIONAL FEE (37 CFR 1.492(A)(1)-(5)):

Search Report has been prepared by the EPO or JPO....................................$860.00

International preliminary examination fee paid to USPTO (37 CFR 1.482) $690.00

No international preliminary examination fee paid to USPTO (37 CFR 1.482) but international search fee paid to USPTO (37 CFR 1.445(a)(2))..........................$710.00

Neither international preliminary examination fee (37 CFR 1.482) nor international search fee (37 CFR 1.445(a)(2)) paid to USPTO..........................$1000.00

International preliminary examination fee paid to USPTO (37 CFR 1.482) and all claims satisfied provisions of PCT Article 33(2)-(4)............ ....$ 100.00

CALCULATIONS PTO USE ONLY

ENTER APPROPRIATE BASIC FEE AMOUNT =	$860	
Surcharge of **$130.00** for furnishing the oath or declaration later than [] 20 [] 30 months from the earliest claimed priority date (37 CFR 1.492(e)).	$0	

CLAIMS	NUMBER FILED	NUMBER EXTRA	RATE		
Total claims	20- 20 =	0	X 18	$0	
Independent claims	5- 3 =	2	X 80	$190	
MULTIPLE DEPENDENT CLAIM (S) (if applicable)			+ $270.00	$0	
TOTAL OF ABOVE CALCULATIONS			=	$190	
[] Applicant claims small entity status See 37 CFR 1 27. The fees indicated above are reduced by 1/2.				$0 -	
SUBTOTAL			=	$1,050	
Processing fee of $130.00 for furnishing the English translation later than [] 20 [] 30 months from the earliest claimed priority date (37 CFR 1.492(f))				$0	
TOTAL NATIONAL FEE			=	$1,050	
Fee for recording the enclosed assignment (37 CFR 1 21(h)) The assignment must be accompanied by an appropriate cover sheet (37 CFR 3.28, 3.31). $40.00 per property.			+	$0	
TOTAL FEES ENCLOSED			=	$1,050	
			Amount to be: refunded	$	
			charged	$	

a. [X]   A check in the amount of $1,050 00 to cover the above fees is enclosed.

b. []   Please charge my Deposit Account No. **23-1123** in the amount of $ to cover the above fees.
A duplicate copy of this sheet is enclosed.

c. [X]   The Commissioner is hereby authorized to charge any additional fees which may be required, or credit any overpayment, to Deposit Account No. **23-1123**. A duplicate copy of this sheet is enclosed.

NOTE: Where an appropriate time limit under 37 C.F.R. 1.494 or 1.495 has not been met, a petition to revive (1.37(a) or (b)) must be filed and granted to restore the application to pending status.

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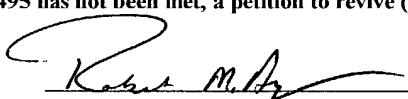
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1

Application Information

Title Line One::	METHOD FOR TRANSMITTING AN OFFSET
Title Line Two::	MODULATED (BFDM/OM) MULTICARRIER SIGNAL
Total Drawing Sheets::	8
Formal Drawings?::	Yes
Application Type::	Utility
Docket Number::	F40.12-0005

Representative Information

Registration Number One::	20,147
Registration Number Two::	24,383
Registration Number Three::	34,557
Registration Number Four::	34,797
Registration Number Five::	34,847
Registration Number Six::	35,612
Registration Number Seven::	36,188
Registration Number Eight::	38,354
Registration Number Nine::	39,758
Registration Number Ten::	41,885
Registration Number Eleven::	42,413

Continuity Information

This application is a::	371
> Application One::	PCT/FR00/02716
Filing Date::	29 September 2000

Prior Foreign Applications

Foreign Application One::	99 12371
Filing Date::	29 September 1999
Country::	France
Priority Claimed::	Yes

IN THE UNITED STATES PATENT AND TRADEMARK OFFICE

First Named Inventor : Pierre Siohan et al.		
Appln. No.:		
Filed : HEREWITH	Group Art Unit:	
For : METHOD FOR TRANSMITTING AN OFFSET MODULATED (BFDM/OM) MULTICARRIER SIGNAL	Examiner:	
Docket No.: F40.12-0005		

PRELIMINARY AMENDMENT

EXPRESS MAIL NO. EV049900628US
DATE OF DEPOSIT: March 26, 2002

Box Non-Fee Amendment
Commissioner for Patents
Washington, D.C. 20231
Sir:

Please amend the above-identified as follows:

IN THE SPECIFICATION

On Page 1, before line 1 and after the Title of the Invention, please insert the following:

CROSS-REFERENCE TO RELATED APPLICATION

This application is a Section 371 National Stage application of International Application No. PCR/FR00/02716 filed September 29, 2000 and published April 5, 2001 as WO 01/24470, not in English.

On Page 1, between lines 6 and 7, please insert the following:

BACKGROUND OF THE INVENTION

On Page 5, between lines 2 and 3, please insert the following:

SUMMARY OF THE INVENTION

Please amend the paragraph beginning on Page 5, line 27

and ending on Page 6, line 10 as follows:

It shall be noted that such a modulated transmultiplexer structure, providing transmission of an offset modulated multicarrier signal is highly different from the structures of prior art transmultiplexers. Indeed, known schemes of transmultiplexers have decimation-expansion factors less than or equal to the number of implemented sub-bands as described, for example, in the textbook "Wavelets and Filter Banks" of G. Strand and T. Nguyen (Wellesley Cambridge Press, Wellesley, MA, USA 1996). On the other hand, with the approach of the invention consisting of implementing on each of the branches of the filter banks, filtering means derived from a predetermined prototype modulation function, a number of sub-bands may be obtained which is larger than (double) the expansion and decimation factor.

On Page 9, between lines 5 and 6, please insert the following:

BRIEF DESCRIPTION OF THE DRAWINGS

On Page 10, between lines 9 and 10, please insert the following:

DETAILED DESCRIPTION OF THE PREFERRED EMBODIMENTS

IN THE CLAIMS

Please cancel claim 10.

Please amend claims 1, 3-4, 6 and 8-9 as follows:

1.(Amended)   A method for transmitting a BFDM/OM biorthogonal multicarrier signal characterized in that it implements a transmultiplexer structure providing:

a modulation step, by means of a bank of synthesis filters, having 2M parallel branches, $M \geq 2$, each fed by source data and each comprising an expander of order M and filtering means;

a demodulation step, by means of a bank of analysis filters, having 2M parallel branches, each comprising a decimator of order M and filtering means, and delivering representative data received from said source data,

said filtering means being derived from a predetermined prototype modulation function.

Claim 2 remains unchanged.

3.(Amended) The transmission method according to claim 2, characterized in that at least one of said polyphase matrices comprises a reverse Fourier transform with 2M inputs and 2M outputs.

4.(Amended) The modulating method according to claim 12, characterized in that it implements a reverse Fourier transform fed by 2M source data, each having undergone a predetermined phase shift, and feeding 2M filtering modules, each followed by an expander of order M, the outputs of which are grouped then transmitted.

Claim 5 remains unchanged.

6.(Amended) The demodulating method according to claim 15, characterized in that it implements a reverse Fourier transform fed by 2M branches, themselves fed by said transmitted signal, and each comprising a decimator of order M followed by a filtering module, and feeding 2M phase shift multipliers, delivering an estimation of the source data.

Claim 7 remains unchanged.

8.(Amended)   The demodulation method according to claim 15, characterized in that said filtering modules are produced as one of the filters belonging to the group comprising:

transverse structure filters;

ladder structure filters; and

trellis structure filters.

9.(Amended)   The modulation method according to claim 15, characterized in that said orthogonal multicarrier signal is a OFDM/OM signal.

Claim 10 has been cancelled.

Please add new claims 11-21 as follows:

11.(New) The method according to claim 1, characterized in that said orthogonal multicarrier signal is an OFDM/OM signal.

12.(New) The method for modulating a BFDM/OM biorthogonal multicarrier signal, characterized in that it implements a bank of synthesis filters having 2M parallel branches, M $\geq$ 2, each fed by source data and each comprising an expander of order M and filtering means, said filtering means being derived from a predetermined prototype modulation function.

13.(New) The modulation method according to claim 12, characterized in that said filtering modules are produced as one of the filters belonging to the group comprising:

transverse structure filters;

ladder structure filters; and

trellis structure filters.

14.(New) The method according to claim 12, characterized in that

said orthogonal multicarrier signal is an OFDM/OM signal.

15. (New) A method for demodulating a BFDM/OM biorthogonal multicarrier signal characterized in that it implements a bank of analysis filters having 2M parallel branches, each comprising an expander of order M and filtering means, and delivering representative data received from source data, said filtering means being derived from a predetermined prototype modulation function.

16. (New) Apparatus comprising:

a modulating device for modulating a BFDM/OM biorthogonal multicarrier signal, characterized by a bank of synthesis filters having 2M parallel branches, $M \geq 2$, each fed by source data and each comprising an expander of order M and filtering means, said filtering means being derived from a predetermined prototype modulation function.

17. (New) The apparatus according to claim 16, wherein the modulating device is further characterized in that it implements a reverse Fourier transform fed by 2M source data, each having undergone a predetermined phase shift, and feeding 2M filtering modules, each following by an expander of order M, the outputs of which are grouped then transmitted.

18. (New) The apparatus according to claim 16, further including a demodulation device for demodulating a BFDM/OM orthogonal multicarrier signal characterized by:

a bank of analysis filters having 2M parallel branches, each comprising an expander of order M and filtering means, and delivering representative data received from source data, said filtering means being derived from a predetermined prototype modulation function.

19. (New) The apparatus according to claim 20, wherein the demodulating device is further characterized in that it implements a reverse Fourier transform fed by 2M branches, themselves fed by said transmitted signal, and each comprising a decimator of order M followed by a filtering module, and feeding 2M phase shift multipliers, delivering an estimation of the source data.

20. (New) A demodulation device for demodulation a BFDM/OM biorthogonal multicarrier signal characterized by:

a bank of analysis filters having 2M parallel branches, each comprising an expander of order M and filtering means, and delivering representative data received from source data, said filtering means being derived from a predetermined prototype modulation function.

21. (New) The demodulation device according to claim 20, further characterized in that it implements a reverse Fourier transform fed by 2M branches, themselves fed by said transmitted signal, and each comprising a decimator of order M followed by a filtering module, and feeding 2M phase shift multipliers, delivering an estimation of the source data.

REMARKS

Favorable action is respectfully requested.

The Director is authorized to charge any fee deficiency required by this paper or credit any overpayment to Deposit Account No. 23-1123.

Respectfully submitted,

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RMA:tas

MARKED-UP VERSION OF REPLACEMENT PARAGRAPHS

It shall be noted that such a modulated transmultiplexer structure, providing transmission of an offset modulated multicarrier signal is highly different from the structures of prior art transmultiplexers. Indeed, known schemes of transmultiplexers have decimation-expansion factors less than or equal to the number of implemented sub-bands as described, for example, in the textbook "Wavelets and Filter Banks" of G. Strand and T. Nguyen (Wellesley Cambridge Press, Wellesley, MA, USA 1996). On the other hand, with the approach of the invention consisting of implementing on each of the branches of the filter banks, filtering means derived from a predetermined prototype modulation function, a number of sub-bands may be obtained which is larger than (double) the expansion and decimation factor.

MARKED-UP VERSION OF REPLACEMENT CLAIMS

1.(Amended)   A method for transmitting a BFDM/OM biorthogonal multicarrier signal characterized in that it implements a transmultiplexer structure providing:

————a modulation step, by means of a bank of synthesis filters——(11), having 2M parallel branches, $M \geq 2$, each fed by source data and each comprising an expander of order M and filtering means;

————a demodulation step, by means of a bank of analysis filters——(12), having 2M parallel branches, each comprising a decimator of order M and filtering means, and delivering representative data received from said source data,

said filtering means being derived from a predetermined prototype modulation function.

Claim 2 remains unchanged.

3.(Amended)   The transmission method according to claim 2, characterized in that at least one of said polyphase matrices comprises a reverse Fourier transform (51,61) with 2M inputs and 2M outputs.

4.(Amended)   AThe modulating method for modulating a signal transmitted according to the method of any of claims 1 to 312, characterized in that it implements a reverse Fourier transform (51) fed by 2M source data, each having undergone a predetermined phase shift, and feeding 2M filtering modules, each followed by an expander of order M, the outputs of which are grouped then transmitted.

Claim 5 remains unchanged.

6.(Amended)   A~~The~~ demodulating method ~~for demodulating a signal transmitted~~ according to ~~the method of any of~~ claims ~~1 to 3~~ 15, characterized in that it implements a reverse Fourier transform ~~(61)~~ fed by 2M branches, themselves fed by said transmitted signal, and each comprising a decimator of order M followed by a filtering module, and feeding 2M phase shift multipliers, delivering an estimation of the source data.

Claim 7 remains unchanged.

8.(Amended)   The ~~modulation method according to any of claims 4 and 5, or the~~ demodulation method according to ~~any of~~ claims ~~6 and 7~~ 15, characterized in that said filtering modules are produced as one of the filters belonging to the group comprising:
    ——transverse structure filters;
    ——ladder structure filters; and
    ——trellis structure filters.

9.(Amended)   The modulation method according to ~~any of~~ claims ~~1 to 8~~ 15, characterized in that said orthogonal multicarrier signal is a OFDM/OM signal.

Claim 10 has been cancelled.

PATENT APPLICATION OF

PIERRE SIOHAN AND CYRILLE SICLET

ENTITLED

METHOD FOR TRANSMITTING AN OFFSET MODULATED (BFDM/OM) MULTICARRIER SIGNAL

METHOD FOR TRANSMITTING A OFFSET MODULATED (BFDM/OM)
MULTICARRIER SIGNAL

The field of the invention is that of the transmission
of digital signals, based on multicarrier modulations. More
specifically, the invention relates to the transmission,
and notably to the modulation and demodulation of
5   biorthogonal multicarrier signals (Biorthogonal Frequency
Division Multiplex/Offset Modulation (BFDM/OM)).

For several years, multicarrier modulations have
aroused large interest. This is in particular justified in
the case of mobile phone communications, where their
10   effectiveness has already been demonstrated for
broadcasting radio signals, with, first of all, the Digital
Audio Broadcasting system (DAB) [1] (for the sake of
simplification and legibility, all references mentioned in
the present description have been grouped in Appendix E)
15   but also in high rate transmissions over telephone two-wire
lines with ADSL (Asymmetric Digital Subscriber Line) and

VDSL (Very high bit rate Digital Subscriber Line) systems [2].

In the usual multicarrier modulation schemes, a set of carrier frequencies selected in order to meet time and frequency orthogonality conditions, is multiplexed. This is the so-called Orthogonally Frequency Division Multiplex (OFDM).

A modulation without any offset (Synchronous Modulation) (SM) or with an offset (Offset Modulation) (OM) may be associated with each of the carriers. This now results in the OFDM/SM and OFDM/OM systems, respectively. In particular, by associating a quadrature amplitude modulation, with or without any offset, with each of the carriers, OFDM/QAM (Quadrature Amplitude Modulation) and OFDM/OQAM (Offset QAM) modulations are produced respectively. This latter modulation operates without any guard interval and also provides a wider possibility of choice as regards the prototype function [3], [4].

However, optimality of OFDM is only ensured by its orthogonality, in the case of transmission channels which may be assimilated with additive white and gaussian noise. In all other cases, OFDM's optimality is not guaranteed.

From this point of view, biorthogonal multicarrier modulations (BFDM) provide further possibilities and in particular they may be a better compromise with regards to mobile radio phone type channels which are dispersive in both time and frequency [5].

Furthermore, with an offset biorthogonal modulation (BFDM/OM), the advantage of OFDM/OM may be retained with the possibility of obtaining prototype functions well localized in time and in frequency.

5   As an indication, a short reminder of the essential definitions relating to the mathematical aspects related to modulations of the BFDM/OM type is given in Appendix A. These aspects have already been the object of publications, with the designation BFDM/OFDM, also retained in the

10   appendices of the present description.

A discretization technique for BFDM/OM modulation systems has already been suggested in a recently submitted article [6]. However, the approach described in [6], [7] is essentially based on discretization of continuous equations

15   which extend the formalism introduced in the continuous domain to the discrete domain, in reference [4] for OFDM/OM.

For OFDM/OM, the use of a mathematical transform and of the reverse transform (conventionally FFT^{-1} then FFT) is

20   therefore assumed. The discretized signal is then truncated.

The object of the invention is notably to provide a new technique for modulating and demodulating a BFDM/OM signal which is more effective and easier to implement as

25   known techniques.

Thus, an object of the invention is to provide such modulation and demodulation techniques which are able to ensure theoretically, that symbol interference (IES) and

channel interference (IEC) are exactly zero, on a finite support.

An object of the invention is also to provide such techniques with which devices may be made which structurally fulfil the cancellation of IES and IEC.

Another object of the invention is to provide such techniques, which provide the implementation of prototype functions, either symmetrical or not and either identical or not, both upon transmission and reception.

Still another object of the invention is to provide such modulation and demodulation techniques with which reconstruction delays may be reduced and controlled, for example for real time or interactive applications. In other words, one object is to provide such techniques with which, for prototype filters of a given length, reconstruction delays may be obtained which are not set (and which may therefore be smaller than those of OFDM/OM).

An object of the invention is also to provide such techniques which are optimum, with respect to distortions, produced by a gaussian channel and/or by non-gaussian channels which are not simply reduced to additive white gaussian noise.

Still another object of the invention, is to provide such techniques, with which higher performances, as compared with known techniques, may be obtained, in terms of localization of the transform.

An object of the invention is also to provide modulation and/or demodulation and more generally devices

for transmitting and/or receiving signals, which are easy and not very expensive to make and implement.

These objects as well as others which will be apparent later on, are achieved according to the invention by means of a method for transmitting a biorthogonal BFDM/OM multicarrier signal, which implements a transmultiplexer structure providing:

- a modulation step, by means of a bank of synthesis filters having 2M parallel branches, $M \geq 2$, each fed with source data and each comprising an expander of order M and filtering means;

- a demodulation step, by means of a bank of analysis filters, having 2M parallel branches, each comprising a decimator of order M and filtering means and delivering representative data received from said source data,

said filtering means being derived from a predetermined prototype modulation function.

In other words, the invention provides a new realization of BFDM/OM modulation systems, based on a novel description of a modulation system, as a transmultiplexer, subsequently called a modulated transmultiplexer. As it will be apparent later on, this technique has many advantages, both in terms of embodiments and effectiveness of the processing operations, and notably for cancelling IES and IEC.

It shall be noted that such a modulated transmultiplexer structure, providing transmission of an

offset modulated multicarrier signal is highly different from the structures of prior art transmultiplexers. Indeed, known schemes of transmultiplexers have decimation-expansion factors more than or equal to the number of

5   implemented sub-bands. On the other hand, with the approach of the invention consisting of implementing on each of the branches of the filter banks, filtering means derived from a predetermined prototype modulation function, a number of sub-bands may be obtained which is larger than (double) the

10 expansion and decimation factor.

Furthermore, as compared with prior art transmultiplexers, such a modulated transmultiplexer structure according to the invention has the advantage of providing a wide selection of prototype filters.

15 Preferentially, said filtering means of said bank of synthesis filters and/or of said bank of analysis filters are grouped as a polyphase matrix, respectively.

Practically, this provides simplification of the operating complexity of the transmultiplexer.

20 Advantageously, at least one of said polyphase matrices comprises a reverse Fourier transform with 2M inputs and 2M outputs. The inventors have actually shown that by using such a transform, for which algorithms are available (IFFT), the realization and the implementation of

25 the invention may be highly simplified.

The invention also relates to the method for modulating a signal transmitted according to the transmission method described above. Such a modulation

method advantageously implements a reverse Fourier transform fed by 2M source data each having undergone a predetermined phase shift and feeding 2M filtering modules, each followed by an expander of order M, the outputs of
5   which are grouped and then transmitted.

The modulation algorithm may then deliver data s[k] such that:

$$x_m^0(n) = a_{m\,n} e^{j\frac{\pi}{2}n}$$

$$x_l^1(n) = \sqrt{2} \sum_{k=0}^{2M-1} x_k^0(n) e^{-j\frac{2\pi}{2M}l\frac{D-M}{2}} e^{j\frac{2\pi}{2M}kl}$$

$$= 2M\sqrt{2}\,\text{IFFT}\left(x_0^0(n), \cdots, x_{2M-1}^0(n) e^{-j\frac{2\pi}{2M}(2M-1)\frac{D-M}{2}} \right)$$

$$x_l^2(n) = \sum_{k=0}^{m-1} p(l+2kM) x_k^1(n-2k)$$

$$s[k] = \sum_{n=\left\lfloor\frac{k}{M}\right\rfloor-1}^{\left\lfloor\frac{k}{M}\right\rfloor} x_{k-nM}^2(n)$$

wherein D = αM − β,
20   with α an integer representing the reconstruction delay;

β an integer between 0 and M−1

and [.] is the "integral part" function.

In the same way, the invention relates to the method
25   for demodulating a signal transmitted according to the transmission method described above. This demodulation

method advantageously implements a reverse Fourier transform fed by 2M branches, themselves fed par said transmitted signal, and each comprising a decimator of order M followed by a filtering module, and feeding 2M phase shift multipliers, delivering an estimation of the source data.

The demodulation method may thus, advantageously, deliver data $a_{m,n-\alpha}$ such that:

$$\hat{x}_l^{.2}(n-\alpha) = s[nM - \beta - l]$$

$$\hat{x}_l^{.1}(n-\alpha) = \sum_{k=0}^{m-1} p(l + 2kM)\hat{x}_l^{.2}(n-\alpha-2k)$$

$$\hat{x}_l^{.0}(n-\alpha) = \sqrt{2}\, e^{-j\frac{2\pi}{2M}l\frac{D+M}{2}} \sum_{l=0}^{2M-1} \hat{x}_l^{.1}(n-\alpha) e^{j\frac{2\pi}{2M}kl}$$

$$= 2M\sqrt{2}\, e^{-j\frac{2\pi}{2M}l\frac{D+M}{2}} \text{IFFT}\left(\hat{x}_l^{'1}(n-\alpha),\cdots,\hat{x}_{2M-1}^{'1}(n-\alpha)\right)$$

$$\hat{a}_{m,n-\alpha} = \Re\left\{ e^{-j\frac{\pi}{2}(n-\alpha)}\hat{x}_l^{.0}(n-\alpha) \right\}$$

Advantageously, in the modulation and/or demodulation method, said filtering modules are produced in one of the forms belonging to the group comprising:

– filters with a transverse structure;

– filters with a ladder structure; and

– filters with a trellis structure.

Other filter structures may of course be contemplated, and notably structures of filters with an infinite impulse response (RII).

According to a particular embodiment, notably corresponding to the trellis structure, said bioctogonal

multicarrier signal is a OFDM/OM signal. Special technical solutions may then be contemplated.

Of course, the invention also relates to the devices for transmitting and/or receiving a BFDM/OM signal, implementing the methods shown above.

Other features and advantages of the invention will become more clearly apparent upon reading the preferred embodiments, given as simple illustrative and non-limiting examples and the appended drawings wherein:

- figure 1 illustrates the general structure of a transmultiplexer associated with BFDM/OM modulation, according to the invention;

- figure 2 in a simplified way, shows a global view of the chain implementing a transmultiplexer such as illustrated in figure 1;

- figure 3 is a representation in a polyphase form of the transmultiplexer of figure 1;

- figure 4 illustrates, on an elementary case, the insertion of a delay operator placed between an expander and a decimator, used in the implementation of the polyphase representation of figure 3;

- figures 5 and 6 respectively show a BFDM/OM modulator and demodulator achieved by means of a reverse FFT;

- figures 7 and 8 show ladder structure filters which may be used instead of the polyphase filters of figures 5 and 6, respectively, when s, an integer parameter defined later on, is even or odd;

- figure 9 illustrates a structure as a trellis for the polyphase filters of figures 5 and 6, in the case of a OFDM/OM signal with symmetrical prototype filters;

- figure 10 shows a trellis according to figure 9, in the normalized case;

- figures 11A and 11B on the one hand and 12A and 12B on the other hand, illustrate the time and frequency responses obtained in two special embodiments, corresponding to the Tables of Appendix D.

As shown earlier, the technique of the invention is notably based on a special approach to discretization, aiming at directly obtaining a description of the modulated transmultiplexer type system. In addition to the advantage of a more general descriptive framework, this approach provides many possibilities for utilizing connections between the banks of filters and the transmultiplexers, for optimizing the realization structures and the computation of the associated coefficients.

After having shown the general structure for representing BFDM/OM systems, as a discrete model of the transmultiplexer type, four special embodiments of the invention are shown hereafter which respectively correspond to:

- two BFDM/OM embodiments which, at the modulator and the demodulator, both use a fast reverse Fourier transform algorithm (IFFT) and differ by the type of implantation of the polyphase components of the prototype filter:

- Mode 1: IFFT algorithm + transverse polyphase filtering;

- Mode 2: IFFT algorithm + ladder filtering.

- Two embodiments adapted to OFDM/OM, derived from BFDM/OM:

- Mode 3: an alternative Mode 1 verifying the discrete orthogonality of OFDM/OM with transverse polyphase filtering and the possibility of implementing a symmetrical prototype filter or not;

- Mode 4: and alternative Mode 2 verifying the discrete orthogonality of OFDM/OM with polyphase filtering achieved by a trellis structure.

Methods for designing prototype filters illustrating these methods for achieving BFDM/OM and OFDM/OM modulations, are also shown.

The results shown notably illustrate:

- the additional possibilities of BFDM/OM, so that the transmission delay remains adjustable for a given prototype filter length. For example, performances in terms of time-frequency localization of the transformation associated to the modulator may be enhanced for an identical transmission delay. With this, high performances may also be maintained from the point of view of selectivity while reducing the transmission delay;

- in the case of so-called back-to-back systems, the possibility with modes 2 and 4 of totally cancelling out the interference between symbols (IES) and the

interference between channels (IEC) and thereby obtaining what may also be called perfect reconstruction.

Other examples not reported here, also show that it is possible to obtain localization performances comparable to those of OFDM/OM, -biorthogonally, and this with much shorter prototype filters. To facilitate interpretation, the following notations are retained: the sets, for example **R** the real number field, as well as vectors and matrices, for example **E**(z) and **R**(z), the polyphase matrices, are marked in bold characters. Otherwise, all the mathematical symbols used are marked in standard characters with generally the time functions in lower case, and .the functions of transformed domains (both z- and Fourier-transformed) are in upper case.

1-Formulation as a modulated transmultiplexer

Starting with a causal prototype filter $p[k]$, derived from $h(t)$ by translation and discretization, we obtain a realization scheme which is the one of figure 1.

In this scheme, filters $F_i[z]$ 11 and $H_i[z]$ 12, with $0 \leq i \leq 2M -1$, are derived from $p[k]$ (or $P(z)$) by complex modulation. α and β, $0 \leq \beta \leq M -1$, are two integers which are related to a parameter D of the modulation $D = \alpha M - \beta$. The calculations by means of which this scheme may be achieved, are reported in Appendix B.

It may also be noted, that the prototype filters may be different. Subsequently, we will merely study the

particular case when $q[k] = p[d-k]$, without this limiting the scope of the patent application.

The realization of a modulation and demodulation scheme directly according to this figure 1 would be extremely costly, in terms of operative complexity. According to the approach of the invention, the prototype filters $P(z)$, are therefore broken down into their polyphase components $G_l(z)$ as shown in Appendix C.

Appendix C also specifies the input/output relationship, the conditions to be observed on the polyphase components and the construction delay.

2-Exemplary embodiments

All the embodiments described later on are based on the implementation of a Discrete Fourier Transform (DFT).

Of course, this technique has the advantage that the DFT is expressed by fast computation algorithms, designated by their acronyms FFT, or IFFT for the inverse transform. (It shall be noted that the referenced equations (1) to (54) are found in Appendices A to C).

Let us write:

$$W_1^{'} = \sqrt{2} \begin{pmatrix} 1 & & & 0 \\ & e^{-j\frac{2\pi}{2M}\frac{D+M}{2}} & & \\ & & \ddots & \\ 0 & & & e^{-j\frac{2\pi}{2M}(2M-1)\frac{D+M}{2}} \end{pmatrix} \tag{55}$$

$$W_2 = \sqrt{2} \begin{pmatrix} 1 & & & 0 \\ & e^{-j\frac{2\pi}{2M}\frac{D-M}{2}} & & \\ & & \ddots & \\ 0 & & & e^{-j\frac{2\pi}{2M}(2M-1)\frac{D-M}{2}} \end{pmatrix} \qquad (56)$$

And W is the discrete Fourier transform of dimensions $2M \times 2M$:

$$[W]_{k,l} = e^{-j\frac{2\pi}{2M}kl}, \quad 0 \le l, k \le 2M-1 \qquad (57)$$

By using equations (35)-(38) (Appendix C), one obtains:

$$R(z^2) = \begin{pmatrix} 0 & & G_{2M-1}(z^2) \\ & \ddots & \\ G_0(z^2) & & 0 \end{pmatrix} W \, W_2 \qquad (58)$$

$$E(z^2) = W_1 \, W^* \begin{pmatrix} G_0(z^2) & & 0 \\ & \ddots & \\ 0 & & G_{2M-1}(z^2) \end{pmatrix} \qquad (59)$$

The schemes of the modulator of figure 5 and of the demodulator of figure 6 are derived from this, both achieved by means of an inverse Fourier transform IFFT 51, 61. In these figures 5 and 6, s is an integer defined by $D = 2.s.M + d$, d being an integer between 0 and 2M-1.

Of course, the notations and data appearing in figures 5 and 6, as well as in the other figures, are a full part of the present description.

For the sake of simplification, but without any loss in generality, it is assumed hereafter that the prototype

filter $P(z)$ has a length of $2mM$ so that all the polyphase components have the same length m.

2.1 Mode 1:IFFT algorithm and breakdown into polyphase components

Again using the notations of figures 5 and 6, the following modulation and demodulation algorithms already mentioned above are derived:

2.1.1 Modulation algorithm

$$x_m^0(n) = a_{m,n} e^{j\frac{\pi}{2}n} \tag{60}$$

$$x_l^1(n) = \sqrt{2} \sum_{k=0}^{2M-1} x_k^0(n) e^{j\frac{2\pi}{2M}k\frac{D-M}{2}} e^{j\frac{2\pi}{2M}kl} \tag{61}$$

$$= 2M\sqrt{2}\,\text{IFFT}\left(x_0^0(n), \cdots, x_{2M-1}^0(n) e^{-j\frac{2\pi}{2M}(2M-1)\frac{D-M}{2}} \right) \tag{62}$$

$$x_l^2(n) = \sum_{k=0}^{m-1} p(l+2kM) x_k^1(n-2k) \tag{63}$$

$$s[k] = \sum_{n=\left\lfloor\frac{k}{M}\right\rfloor-1}^{\left\lfloor\frac{k}{M}\right\rfloor} x_{k-nM}^2(n) \tag{64}$$

2.1.2 Demodulation algorithm

$$\hat{x}_l^2(n-\alpha) = s[nM - \beta - l] \tag{65}$$

$$\hat{x}'^1_l(n-\alpha) = \sum_{k=0}^{m-1} p(l+2kM)\hat{x}'^2_l(n-\alpha-2k) \qquad (66)$$

$$\hat{x}^0_l(n-\alpha) = \sqrt{2}e^{-j\frac{2\pi}{2M}l\frac{D+M}{2}}\sum_{k=0}^{2M-1}\hat{x}'^1_l(n-\alpha)e^{j\frac{2\pi}{2M}kl} \qquad (67)$$

$$= 2M\sqrt{2}e^{-j\frac{2\pi}{2M}l\frac{D+M}{2}}\mathrm{IFFT}\left(\hat{x}'^1_l(n-\alpha),\cdots,\hat{x}'^1_{2M-1}(n-\alpha)\right) \qquad (68)$$

$$\hat{a}_{m\;n-\alpha} = \Re\left\{e^{-j\frac{\pi}{2}(n-\alpha)}\hat{x}^0_l(n-\alpha)\right\} \qquad (69)$$

2.2 Mode 2: IFFT and ladder structure

Ladder schemes are a implementation means recently
20 suggested for producing banks of filters. The inventors
have mathematically validated their application to BFDM/OM,
as described hereafter.

It is seen that a BFDM/OM modulation may be written as
a transmultiplexer using two inverse FFTs (figures 5 and
25 6), wherein the polyphase components of the used prototype
appear explicitly. Each polyphase filter may then be
written as a ladder. According to whether s is even or odd,
filters $G_l(z)$ of figures 5 and 6 may be replaced with the
schemes given by figures 7 and 8.

30 In order to achieve such schemes, a matrix breakdown
of the polyphase components is implemented, which is based
on 2x2 matrices, the number and nature of which are
determined according to the desired prototype length and
reconstruction delay.

For example, in order to generate the pair of polyphase components $[G_1(z), G_{M+1}(z)]$, we proceed in two steps:

- initialization is performed by a couple $(\boldsymbol{F}_0, \boldsymbol{F}_1)$. $\boldsymbol{F}_0$ corresponds to a product of three matrices which will match the first three items of the upper schemes in figures 7 and 8. The exact form of $\boldsymbol{F}_1$ depends on the parity of parameter s. This is the identity matrix for even s (cf. upper scheme in figure 7) or this is a product of two matrices C_0 and B_0 for odd s, which will match the following two items in the upper scheme in figure 8. A prototype of length $2M$ (s=0) or $4M$ (s=1) is thereby obtained.

- In order to increase the length of $P(z)$, with or without increasing the delay, a set of matrices are then applied, which will be either $(\boldsymbol{A}, \boldsymbol{B})$, or $(\boldsymbol{C}, \boldsymbol{D})$ respectively. The continuation of the embodiment scheme is thereby obtained.

The same principle is applied to the polyphase components $[G_{d-1}(z), G_{d-M-1}(z)]$, this time taking the inverse matrices of the previous matrices.

An advantage of this structure is that it guarantees perfect reconstruction even in the presence of an error on the calculated coefficients, in particular in the presence of quantification errors.

Furthermore, this structure also facilitates optimization of the prototype filter, for example by considering a localization or frequency selectivity

criterion: it is sufficient to optimize $([(M-1)/2]+1)(2m+1)-mM$ coefficients instead of $2mM$, without introducing any perfect reconstruction constraint.

5   3- Complexity of the various embodiments

In order to perform a comparison of the different embodiments provided, we shall place ourselves in the common case where $N = 2mM$. In this case each polyphase component has a length equal to m.

10   Each polyphase component may be produced as a transverse component, as a ladder component or in the orthogonal case, as a trellis component. Even if the ladders and the trellises have two outputs, only one may be exploited.

15   On each sub-band, the following operations are performed at the modulator:

– premodulation (a phase shift, i.e. a complex multiplication);

– an inverse Fourier transform;

20   – polyphase filtering.

At the demodulator, the same operations are performed in the reverse order. Therefore, the complexity of the complete transmultiplexer with premodulation may be derived in terms of complex operations (Table 1) or real operations

25   (Table 2).

TABLE 1 - Number of complex operations per sub-band and per sample for the full transmultiplexer

	Complex additions	Complex multiplications
Transverse realization	$2m - 2 + 2\log_2 2M$	$2m + 2 + 2\log_2 2M$
Ladder realization	$4m + 2 + 2\log_2 2M$	$4m + 2 + 2\log_2 2M$
Trellis realization (normalized)	$4m - 4 + 2\log_2 2M$	$4m + 2 + 2\log_2 2M$

TABLE 2 - Number of real operations per sub-band and per sample for the modulator (or the demodulator)

	Real additions	Real multiplications
Transverse realization	$2m + 3\log_2 2M$	$2m + 4 + 2\log_2 2M$
Ladder realization	$4m + 4 + 3\log_2 2M$	$4m + 6 + 2\log_2 2M$
Trellis realization (normalized)	$4m + 3\log_2 2M$	$4m + 4 + 2\log_2 2M$

The achieved gain with respect to direct realization of the scheme of figure 1 is therefore a net gain, as the latter would require $2mM - 1$ complex additions and $2mM + 1$ complex multiplications per sub-band and per sample, at both the modulator and the demodulator.

In terms of memory cells, 4M complex values must be stored in order to perform the premodulation, as well as the coefficients of the various structures. When the same filters upon transmission and reception are used, the first column of Table 3 is obtained. Moreover, in all cases,

4(m+1)M complex values must be stored in a buffer for the polyphase filtering both at the modulator and the demodulator.

5   TABLE 3 - Real memory cells for the full modulator (or demodulator)

	ROM	RAM
Transverse realization	$2(m + 1)$ $M + 2$	$4(m + 3)M$
Ladder realization	*(2m+1)[((M-1)/2)+2M + 2]*	$4(m + 3)M$
Symmetrical transverse filter	$(m + 2)M + 2$	$4(m + 3)M$
Trellis realization (normalized)	$m[M/2] + 2M + 2$	$4(m + 3)M$

The different techniques provided are notably characterized by the fact that for a modulator-demodulator

10   system put "back-to-back", their IESs and IECs are exactly zero. Practically, because of the inaccuracy of the numerical calculation, they are generally of the order of 10^{-14}.

In the case of modes 2 and 4, this perfect

15   reconstruction characteristic is provided structurally, i.e. it is maintained after quantification of the ladder coefficients for BFDM/OM or trellis coefficients for OFDM/OM.

Two criteria may be taken into account for designing

20   the prototype filters: localization and selectivity. Other aspects may also be taken into account, such as

representative channel distortions of different transmission channels, for example of the mobile radio type.

As purely indicative examples, Tables 4 and 5 of the Appendix D give particular embodiments of the invention, the results of which are illustrated by figures 11A, 11B, 12A and 12B.

Figures 11A and 11B show the time response and the frequency response for a biorthogonal prototype with $M = 4$, $N = 32$, $\alpha = 8$, $\xi = 0.9799$ (localization), $\xi_{mod} = 0.9851$ (modified localization, according to Doroslovacki's criterion). They match the first column of Table 4 (transverse coefficients) and Table 5 (ladder coefficients).

Figures 12A and 11B respectively show the time response and the frequency response for a biorthogonal prototype with $M = 4$, $N = 32$, $\alpha = 2$, $\xi = 0.9634$ (localization), $\xi_{mod} = 0.9776$ (modified localization, according to Doroslovacki's measurement). They match the second column of Table 4.

APPENDIX A

BFDM/OQAM type multicarrier modulation

In this appendix, as an introduction to BFDM/OQAM modulations, a few essential definitions on biorthogonality ([16], [17], [18]) will be given as a reminder

Let $\mathbf{E}$ be a vector space on a field $\mathbf{K}$, the definitions and properties which we are going to use for generating a BFDM/OQAM modulation, may be summarized in the following way:

Definition A.1

Let $(x_i)_{i \in I}$ and $(\tilde{x}_i)_{i \in I}$ be two families of $\mathbf{E}$ vectors. $(x_i)_{i \in I}$ and $(\tilde{x}_i)_{i \in I}$ are biorthogonal if and only if $\forall (i,j) \in I^2, \left\langle x_i, \tilde{x}_j \right\rangle = \delta_{i,j}$

Definition A.2

Let $(x_i)_{i \in I}$ and $(\tilde{x}_i)_{i \in I}$ be two families of $\mathbf{E}$ vectors. $(x_i)_{i \in I}$ and $(\tilde{x}_i)_{i \in I}$ form a pair of biorthogonal bases of $\mathbf{E}$ if and only if :

- *$(x_i)_{i \in I}$ and $(\tilde{x}_i)_{i \in I}$ form two bases of $\mathbf{E}$*

- *$(x_i)_{i \in I}$ and $(\tilde{x}_i)_{i \in I}$ are two biorthogonal families*

Property A.1

Let $((x_i)_{i \in I}, (\tilde{x}_i)_{i \in I})$ be a pair of orthogonal bases of E, then $\forall x \in \mathbf{E}$:

$$x = \sum_{i \in I} \langle x_i, x \rangle \tilde{x}_i = \sum_{i \in I} \langle \tilde{x}_i, x \rangle x_i$$

$$x = \sum_{i \in I} \alpha_i x_i, \ alors \ \alpha_i = \langle \tilde{x}_i, x \rangle$$

$$x = \sum_{i \in I} \tilde{\alpha}_i \tilde{x}_i, \ alors \ \tilde{\alpha}_i = \langle x_i, x \rangle$$

- $\|x\|^2 = \sum_{i \in I} \langle x_i, x \rangle^* \langle \tilde{x}_i, x \rangle$

A frequency-modulated complex signal on $2M$ sub-carriers may be written as :

$$s(t) = \sum_{n=-\infty}^{+\infty} \sum_{m=0}^{2M-1} a_{m,n} h(t - n\tau_0) e^{2j\pi(f_0 + m\nu_0)t} e^{j\varphi_{m,n}} \qquad (1)$$

with:

-     $a_{m,n} \in \mathbf{R}$;

-     h is a real prototype filter with a bandwidth ν_0 and a finite support: h(t) $\in$ $[-T_1, T_2]$ with T_1 and T_2 real numbers;

-     $f_0 = 0$:

-     $\nu_0 \tau_0 = \frac{1}{2}$.

In order to obtain a biorthogonal modulation, we try to express s(t) with a couple of $\left(\chi_{m,n}, \tilde{\chi}_{m,n} \right)$ biorthogonal bases:

$$s(t) = \sum_{n=-\infty}^{+\infty} \sum_{m=0}^{2M-1} a_{m,n} \chi_{m,n}(t) \qquad (2)$$

with:

$$a_{m,n} = \Re \left\{ \int_{-\infty}^{+\infty} s(t) \tilde{\chi}_{m,n}^*(t) dt \right\} = \langle s, \tilde{\chi}_{m,n} \rangle \qquad (3)$$

Derivation of the expression of the associated discrete bases is shown is Appendix 2.

After translation by T_1 and discretization with a period $T_e = \tau_0/M = 1/2M\nu_0$, it is also possible, cf. Appendix

5   2, to define a pair of discrete biorthogonal bases such as :

$$s[k] = \sum_{n=-\infty}^{+\infty} \sum_{m=0}^{2M-1} a_{m,n} \chi_{m,n}[k] \qquad (4)$$

$$a_{m,n} = \Re\left\{ \sum_{k} s[k]\tilde{\chi}_{m,n}^{*}[k] \right\} = \langle s, \tilde{\chi}_{m,n} \rangle \qquad (5)$$

with

$$\chi_{m,n}[k] = (-1)^{mn}\sqrt{2}\ p[k-nM]e^{j\frac{2\pi}{2M}m(k-nM-\frac{D-M}{2})}$$
$$\tilde{\chi}_{m,n}[k] = (-1)^{mn}\sqrt{2}\ q[k-nM]c^{j\frac{2\pi}{2M}m(k-nM-\frac{D-M}{2})} \qquad (6)$$

$$(7)$$

APPENDIX B

The BFDM/OQAM transmultiplexer

5   General biorthogonal case

Let N be the length of the prototype filter $p[k]$, such that:

$$2T = T_1 + T_2 = (N-1)T_e \tag{8}$$

and $T_1 = 2\lambda T$, $T_2 = 2(1-\lambda)T$ with $\lambda \in [0,1]$. Then:

$$
\begin{aligned}
s[k] &= \sqrt{2} \sum_{n=-\infty}^{+\infty} \sum_{m=0}^{2M-1} a_{m,n}(-1)^{mn} p[k-nM] \\
&\qquad \times e^{j\frac{2\pi}{2M}m(k-nM)} e^{j\left(\varphi_{m,n} - \frac{2\pi}{2M}m\lambda(N-1)\right)}
\end{aligned}
\tag{9}
$$

$$
\begin{aligned}
&= \sqrt{2} \sum_{n=-\infty}^{+\infty} \sum_{m=0}^{2M-1} a_{m,n}(-1)^{mn} p[k-nM] \\
&\qquad \times e^{j\frac{2\pi}{2M}m\left(k-nM-\frac{D-M}{2}\right)} e^{j\left(\varphi_{m,n} + \frac{2\pi}{2M}m\left(\frac{D-M}{2} - \lambda(N-1)\right)\right)}
\end{aligned}
\tag{10}
$$

15   with D an arbitrarily set parameter and with which, as it will be seen, the reconstruction delay may be handled. Considering equation (10), let us now set:

$$\varphi_{m,n} = n\frac{\pi}{2} - \frac{2\pi}{2M}\left(\frac{D-M}{2} - \lambda(N-1)\right) \tag{11}$$

$$x_m(n) = (-1)^{mn} e^{j\frac{\pi}{2}n} a_{m,n} \tag{12}$$

$$f_m(k) = \sqrt{2}\, p[k] e^{j\frac{2\pi}{2M}m\left(k - \frac{D-M}{2}\right)} \tag{13}$$

so that

$$s[k] = \sum_{n=-\infty}^{+\infty} \sum_{m=0}^{2M-1} x_m(n) f_m(k-nM) \tag{14}$$

Moreover, the demodulation dual base is written as:

$$\tilde{\lambda}_{m,n}[k] = (-1)^{mn}\sqrt{2}\, q[k - nM]e^{j\frac{2\pi}{2M}m\left(k - nM - \frac{D-M}{2}\right)}e^{j\frac{\pi}{2}n}$$

(15)

$$\hat{a}_{m,n} = \Re\left\{(-1)^{mn}\sqrt{2}\sum_k s[k]q[k - nM]\right.$$

$$\left. \times e^{-j\frac{2\pi}{2M}m\left(k - nM - \frac{D-M}{2}\right)}e^{-j\frac{\pi}{2}n}\right\}$$

(16)

$$= \Re\left\{(-1)^{mn}\sqrt{2}\sum_k s[D + nM - k]q[D - k]\right.$$

(17)

$$\left. \times e^{-j\frac{2\pi}{2M}m\left(D - k - \frac{D-M}{2}\right)}e^{-j\frac{\pi}{2}n}\right\}$$

$$= \Re\left\{(-1)^{mn}\sqrt{2}\sum_k s[D + nM - k]q[D - k]\right.$$

(18)

$$\left. \times e^{j\frac{2\pi}{2M}m\left(k - \frac{D+M}{2}\right)}e^{-j\frac{\pi}{2}n}\right\}$$

which leads us to put:

$$h_m(k) = \sqrt{2}\, q[D - k]e^{j\frac{2\pi}{2M}m\left(k - \frac{D+M}{2}\right)}$$

(19)

and

$D = \alpha M - \beta$ with α and β integers and $0 \leq \beta \leq M - 1$ (20)

so that

$$\hat{a}_{m,n-\alpha} = \Re\left\{(-1)^{m(n-\alpha)}e^{-j\frac{\pi}{2}(n-\alpha)}\sum_k s[nM - k - \beta]h_m(k)\right\}$$

(21)

The factor $(-1)^{mn}$ appears both in the modulator and the demodulator so it may be deleted without changing anything, and we are then led to the multiplexer scheme of figure 1.

Special orthogonal case

In the orthogonal case, we have $D = N - 1$ and $q[k] = p[k]$, hence:

$$f_m(k) = \sqrt{2}p[k]e^{j\frac{2\pi}{2M}m(k-\frac{N-1}{2})}e^{j\frac{\pi}{2}m} \tag{22}$$

$$h_m(k) = \sqrt{2}p[N-1-k]e^{j\frac{2\pi}{2M}m(k-\frac{N-1}{2})}e^{-j\frac{\pi}{2}m} \tag{23}$$

If $q[k] = p[D-k]$, we then have $q[k] = p[N-1-k]$; the prototype is symmetrical. However, unlike what may often be read implicitly or explicitly ([4], [6], [7], [9]), the symmetry of the prototype is absolutely not required. To persuade ourselves that this is the case, we shall take one of the following prototypes and numerically check out (a direct check is rather tedious) that it provides perfect reconstruction for $M = 4$ in the orthogonal case (we then have N − 1 = D = 7, $\alpha = 2$ and $\beta = 1$):

$$P(z) = \frac{1}{4}\left(1 + z^{-1} + z^{-2} + z^{-3} - z^{-4} - z^{-4} - z^{-5} - z^{-6} - z^{-7}\right) \tag{24}$$

$$P(z) = \frac{1}{4}\left(1 + z^{-1} - z^{-2} + z^{-3} + z^{-4} + z^{-4} + z^{-5} - z^{-6} + z^{-7}\right) \tag{25}$$

It may even be checked that any prototype

$$P(z) = \sum_{n=0}^{\gamma} p(n)z^{-n}$$

which verifies (26) to (31) also provides perfect reconstruction for $M = 4$ in the orthogonal case:

$$c_0 = \pm 1 \ , \quad c_1 = \pm 1 \tag{26}$$

$$\varepsilon_0 = \pm 1 \ , \quad \varepsilon_1 = \pm 1 \tag{27}$$

$$|p(0)| \leq \frac{\sqrt{2}}{4} \ , \quad |p(1)| \leq \frac{\sqrt{2}}{4} \tag{28}$$

$$p(2) = \varepsilon_1 \sqrt{\frac{1}{8} - p(1)^2} \quad , \quad p(3) = \varepsilon_0 \sqrt{\frac{1}{8} - p(0)^2} \tag{29}$$

$$p(4) = c_0 \varepsilon_0 \sqrt{\frac{1}{8} - p(0)^2} \quad , \quad p(5) = c_1 \varepsilon_1 \sqrt{\frac{1}{8} - p(1)^2} \tag{30}$$

$$p(6) = c_1 \varepsilon_1 p(1) \quad , \quad p(7) = c_0 \varepsilon_1 p(0) \tag{31}$$

APPENDIX C

The biorthogonality condition

5   *The polyphase approach*

Achieving modulation and demodulation schemes according to figure 1 would be extremely costly in terms of operating complexity. By breaking down the prototype $P(z)$ into its polyphase components $G_l(z)$, such as

$$P(z) = \sum_{l=0}^{2M-1} z^{-l} G_l(z^{2M}) \tag{32}$$

it is then possible to express the analysis and synthesis banks as:

$$H_m(z) = \sqrt{2} \sum_{l=0}^{2M-1} e^{\frac{j\pi}{M} m\left(l - \frac{D-M}{2}\right)} z^{-l} G_l(z^{2M}) \tag{33}$$

$$F_m(z) = \sqrt{2} \sum_{l=0}^{2M-1} e^{\frac{j\pi}{M} m\left(l - \frac{D-M}{2}\right)} z^{-l} G_l(z^{2M})$$

$$\tag{34}$$

20

From this, the expression for the polyphase matrices $R(z)$ and $E(z)$ of the banks of filters of the modulator and demodulator is thereby derived:

$$E(z^2) = W_1 \begin{pmatrix} G_0(z^2) & & 0 \\ & \ddots & \\ 0 & & G_{2M-1}(z^2) \end{pmatrix} \tag{35}$$

$$R(z^2) = J \left[W_2 \begin{pmatrix} G_0(z^2) & & 0 \\ & \ddots & \\ 0 & & G_{2M-1}(z^2) \end{pmatrix} \right]^T$$

$$
= \begin{pmatrix} 0 & & G_{2M-1}(z^2) \\ & \cdot^{\cdot^{\cdot}} & \\ G_0(z^2) & & 0 \end{pmatrix} W_2^T \qquad (36)
$$

where J, W_1 and W_{12}

are defined as below:

$$
J = \begin{pmatrix} 0 & & 1 \\ & \cdot^{\cdot^{\cdot}} & \\ 1 & & 0 \end{pmatrix}
$$

$$
[W_1]_{k,l} = \sqrt{2}\, e^{\frac{j\pi}{M}k(l-\frac{D+M}{2})} \qquad (37)
$$

$$
[W_2]_{k,l} = \sqrt{2}\, e^{\frac{j\pi}{M}k(l-\frac{D-M}{2})} \qquad (38)
$$

15   In order to isolate the transmultiplexer function, we introduce the following notation, wherein for convenience in writing, we shall no longer take $x_m(n) = j^n a_{m,n}$, but $x_m(n) = a_{m,n}$, in order to have:

$$
\begin{cases} x_m(n) = a_{m,n} \\ \Re(\hat{x}'_m(n)) = \hat{x}_m(n) = \hat{a}_{m,n} \end{cases} \Rightarrow \begin{cases} X_m(-jz) \xleftrightarrow{TZ} j^n a_{m,n} \\ \hat{X}'_m(-jz) \xleftrightarrow{TZ} j^n \hat{x}'_m(n) \end{cases} \quad (39)
$$

$\leftrightarrow$ means related through a z-transform

$x_m(n)$ represents the real symbols to be
25   transmitted and $\hat{x}'_m(n)$ represents the complex symbols received after extraction of the real part. Figure 2 gives a global view of the chain.

With the polyphase matrices $E(z^2)$ and $R(z^2)$, a polyphase form representation of the transmultiplexer (figure 3) may be obtained. Then, all we have to do is to take the real part of the output samples in $\hat{x}'_m(n-\alpha)$

5 order to reconstruct the input with a delay of α samples.

Input/output relationship

Let us note as $X(z)$, the vector which represents the

10 transmitted data in the z-transformed domain. Upon reception, after demodulation, let us note as $\hat{X}'(-jz)$ the z-transform vector associated with the received data. Extraction of the real part then provides vector $X(z)$. Now,

15 our goal is:

– to determine the input/output relationship, i.e. the relationship between $X(z)$ and $\hat{X}(z)$;

20 – to determine the conditions on the polyphase components $G_l(z)$ of $P(z)$ for guaranteeing the equality:

$$\hat{X}(z) = X(z);$$

– to derive from the latter the construction

25 delay α.

The 3 main items of this scheme for determining the input/output relationship are 2 polyphase matrices $E(z)$ and $R(z)$ as well as the transfer matrix $\Delta_\beta(z)$, connected to the expanders, delays and decimators. To determine the latter,

30 the elementary case illustrated in figure 4 may serve as a basis, for which the transfer function is given by

$$V(z) = \begin{cases} 0 & \text{if } K \text{ is not a multiple of } M \\ z^{-\frac{K}{M}} U(z) & \text{if } K \text{ is a multiple of } M \end{cases}$$

From figure 3, we then have:

$$z^{-\alpha}\,\hat{X}'(-jz) = E(z^2)\Delta_\beta(z)R(z^2)X(-jz)$$

$$(jz)^{-\alpha}\,\hat{X}'(z) = \bar{E}(-z^2)\Delta_\beta(jz)R(-z^2))X(z)$$

$$\hat{X}'(z) = j^\alpha z^\alpha W_1 G(jz)W_2^T X(z) \tag{41}$$

where matrix G(z) is defined below:

$$G(z) = \begin{pmatrix} G_0(z^2) & & 0 \\ & \ddots & \\ 0 & & G_{2M-1}(z^2) \end{pmatrix} \Delta_\beta(z) \begin{pmatrix} & & 0 & & G_{2M-1}(z^2) \\ & & \ddots & \\ & G_0(z^2) & & 0 \end{pmatrix} \tag{42}$$

We then have:

$$\hat{X}(z) = Q(z)X(z) \tag{43}$$

with :

$$Q(z) = \Re\left\{(jz)^\alpha W_1 G(jz)W_2^T\right\} \tag{44}$$

20   After a calculation, we obtain

$$Q(z) = \begin{pmatrix} Q_0(z) & 0 & Q_1(z) & \cdots & Q_{M-1}(z) & 0 \\ 0 & Q_0(z) & 0 & Q_1(z) & & Q_{M-1}(z) \\ Q_1(z) & 0 & Q_0(z) & \ddots & \ddots & \vdots \\ \vdots & Q_1(z) & \ddots & \ddots & 0 & Q_1(z) \\ Q_{M-1}(z) & & \ddots & 0 & Q_0(z) & 0 \\ 0 & Q_{M-1}(z) & \cdots & Q_1(z) & 0 & Q_0(z) \end{pmatrix} \tag{45}$$

with :

$$Q_\xi(z) = 2(-1)^\xi \sum_{l=0}^{M-1} U_l\left(-z^2\right) \cos\left[\frac{2\pi}{M}\xi\left(l - \frac{d}{2}\right)\right] \qquad (46)$$

5   The meaning of d is specified later on. The exact expression for $U_l(-z^2)$ further depends on this parameter d (positive or zero integer), it may then be proved that perfect reconstruction is obtained if and only if:

   − si $0 \leq d \leq M - 1$:

10   − if $0 \leq l \leq d$:

$$G_l(z)\, G_{d-l}(z) + z^{-1}G_{M+l}(z)\, G_{M+d-l}(z) = \frac{z^{-s}}{2M} \qquad (47)$$

   − si $d + 1 \leq l \leq M - 1$:

$$G_l(z)\, G_{2M+d-l}(z) + G_{M+l}(z)\, G_{M+d-l}(z) = \frac{z^{-(s-1)}}{2M} \qquad (48)$$

   − if M ≤ d ≤ 2M−1 :

      − if $0 \leq l \leq d - M$:

$$G_l(z)\, G_{d-l}(z) + G_{M+l}(z)\, G_{d-M-l}(z) = \frac{z^{-s}}{2M} \qquad (49)$$

      − if $d + 1 - M \leq l \leq M - 1$:

$$G_l(z)\, G_{d-l}(z) + z^{-1}G_{M+l}(z)\, G_{M+d-l}(z) = \frac{z^{-s}}{2M} \qquad (50)$$

with d and s, integers defined by D = 2sM + d, s > 0 and $0 \leq d \leq 2M-1$. The reconstruction delay α is related to parameter s by the relationships:

$$\alpha = \begin{cases} 2s & \text{if } d = 0 \\ 2s+1 & \text{if } d \in \{1,...,M\} \\ 2(s+1) & \text{if } d \in \{M+1,...,2M-1\} \end{cases} \qquad (51)$$

The special orthogonal case may be derived from this result, for which $D = N - 1$, with N the length of the paraunitary prototype filter, i.e. a symmetrical filter here. $P(z)$ is said to be paraunitary if :
$$P(z) = z^{-(N-1)}\widetilde{P}(z) \quad \text{with } \widetilde{P}(z) = P*(z^{-1})$$

Indeed, it may verified that:

$$G_{d-l}(z) = z^{-s}\hat{G}_l(z) \qquad \text{si } 0 \leq l \leq d \qquad (52)$$
$$G_{2M+d-l} = z^{-(s-1)}\hat{G}_l(z) \qquad \text{si } d+1 \leq l \leq 2M-1 \qquad (53)$$

Thus, in the special orthogonal case, we have perfect reconstruction with a delay $\alpha = \frac{N-1+\beta}{M}$, if and only if:

$$G_l(z)\,\tilde{G}_l(z) + G_{l+M}(z)\,\tilde{G}_{M+l}(z) = \frac{1}{2M} \qquad 0 \leq l \leq M-1 \qquad (54)$$

Appendix D

Coefficients of the prototype filters obtained by optimization

5

TABLE 4 – Biorthogonal prototypes with $M = 4$ and $N = 32$
(transverse coefficients)

n	Examples from figures 11A, 11B	Examples from figures 12A, 12B
0	-4.460868105953324e-05	5.014949968230972e-02
1	-8.827698704913472e-05	1.455583816489019e-01
2	1.816721975145588e-04	2.500737066757044e-01
3	2.302861759368111e-04	3.228805982747062e-01
4	-1.172447599636272e-03	3.661515202615520e-01
5	-1.827055790732281e-03	3.515099229023638e-01
6	-3.760044826875730e-03	2.545572622632939e-01
7	-6.052599354553620e-03	1.351315191913547e-01
8	-6.541334278009250e-03	5.168757567424829e-02
9	-2.320957844665448e-03	1.125177964848224e-02
10	1.063973252261601e-02	-4.802381010162210e-03
11	4.151536856291601e-02	-1.106221296463278e-02
12	1.043838059706333e-01	-8.872655589434630e-03
13	2.005189128209921e-01	-3.754266678003194e-03
14	2.913131449163113e-01	-1.654867643757010e-03
15	3.352627462532674e-01	-1.383187971152199e-03
16	3.351172696026857e-01	-3.813932123570836e-04
17	2.909415993397522e-01	5.624569918905843e-06
18	2.000454638421703e-01	-2.475635347949224e-06
19	1.039959799288574e-01	4.952234305253537e-05
20	4.124129545474275e-02	2.048787314180216e-05
21	1.040407270162191e-02	1.004915628115845e-07
22	-2.440939909195805e-03	4.423431446835775e-08
23	-6.649467765409857e-03	2.649149072234273e-06
24	-5.878063999471562e-03	1.785190585201287e-08
25	-2.983359331348727e-03	6.863128678632993e-12
26	-2.475652683945518e-03	-3.021006714034441e-12
27	-8.249232623G23326e 01	-2.308376067571482e-09
28	-8.891453128240245e-05	6.598349790230041e-12
29	-4.464223698699074e-04	-5.7894922?21166598e-16
30	3.704504269829711e-04	-2.548414264695020e-16
31	1.247820119405080e-05	8.532126971482393e-13

TABLE 5 – Biorthogonal prototype (ladder coefficients) with

$M = 4$ and $N = 32$ and $\alpha = 8$ (cf. figures 11A, 11B)

	l=0	l=1
$f_{0,0}$	2.820843813510179e−01	1.299280891559943e−01
$f_{0,1}$	−5.673498928902276e−01	−3.310904763283146e−01
$f_{0,2}$	3.721645241266496e−01	−3.738170157940610e−02
$c^0_{,0}$	−1.167480680205269e−01	−2.847704852297620e−02
c^0_{n}	1.064716331427927e−01	−1.181506919769764e−01
a^1_{0}	−1.596131503239696e−01	9.127789670781582e−02
b^1_{0}	−2.626782049187459e+01	−2.054722686392923e+01
c^1_{0}	−1.614470454625511e−04	−3.551599154933042e−04
d^1_{0}	2.628294699122717e+01	2.069686434312222e+01

Appendix E

References

[1] D. Pommier and Yi. Wu. Interleaving of spectrum-spreading in digital radio intended for vehicles. *EBU Rev.-Tech., (217):128-142,* June 1986.

[2] L. Vandendorpe. Fractionnally spaced linear and decision feedback detectors for transmultiplexers. *IEEE Transactions on Signal Processing,* 46(4):996-1011, 1998.

[3] B. Le Floch, M. Alard, and C. Berrou. Coded Orthogonal Frequency Division Multiplex. *Proceedings of the IEEE,* 83:982-996, June 1995.

[4] A. Vahlin and N. Holte. Optimal finite duration pulses for OFDM. *IEEE Trans. Communications, 44(1):10-14, January 1996.*

[5] W. Kozek, A. F. Molish, and E. Bonek. Pulse design for robust multicarrier transmission over doubly dispersive channels. In *Proc. Int. Conf. on Telecommunications (Porto Carras, Greece),* volume 2, pages 313-317, June 1998.

[6] H. Boelcskei, P. Duhamel, and R. Hleiss. A design of pulse shaping OFDM/OQAM systems for wireless communications with high spectral efficiency. *Submitted to IEEE Transactions on Signal Processing, November 1998*

[7]   H. Boelcskei, P. Duhamel, and R. Hleiss, "Design   of
        pulse shaping OFDM/OQAM systems for high data-rate
        transmission   over   wireless   channels". In   *Proc.*
        International   Conference   on   Communications   (ICC);
  5   *Vancouver, June_1999.*

[8]   N. Lacaille. Relations bancs de filtres-modulations
        multiporteuses   et   application   à   l'OFDM/OQAM.
        Technical   report,   DEA,   Université   de   Rennes   I,
        CNET/DMR, 1998.

 10   [9]   R. D. Koilpillai and P. P. Vaidyanathan. "Cosine-
        modulated   FIR   filter   banks   satisfying   perfect
        reconstruction".   *IEEE   Transactions   on   Signal*
        Processing, 40(4):770-783, April 1992.

[10] T. Q. Nguyen and R. D. Koilpillai. The theory and
 15   design of arbitrary length cosine-modulated filter
        banks and wavelets satisfying perfect reconstruction.
        IEEE   Trans.   Signal   Processing,   SP-44(3):473-483,
        March *1996.*

[11] Jalali Ali.   *"Etude   et   architecture   d'un   modem*
 20   *numérique destiné aux modulations multiporteuses*
        de densité 2". PhD thesis, Université de Rennes I,
        France, 1998.

[12] L. C. Calvez and P. Vilbé. On the uncertainty principle
        in discrete signals. *IEEE Trans.* Circuits and Systems-
 25   II, 39(6):394-395, June 1992.

[13] M. I. Doroslovacki Product of second *moments in time*
        and frequency for: discrete-time signals and the

uncertainty *limit. Signal Processing, 67(1),;* May 1998.

[14] C. Roche and P. Siohan. A family of Extended Gaussian Functions with a nearly optimal localization property. In *Proc. First Int. Workshop on Multi-Carrier Spread-Spectrum (Oberpfaffenhofen, Germany),* pages 179- 186, April 1997.

[15] P. Heller, T. Karp, and T. Q. Nguyen. A general formulation of modulated filter banks. *Submitted to IEEE Transactions on Signal Processing* 1996:

[16] M. Vetterli and *J. Kovacevic "Wavelets and Subband Coding". Prentice* Hall, 1995.

[17] H.G. Feichtinger et al. *Gabor Analysis and Algorithm – Theory and Applications.* Birkhäuser, Boston-Basel-Berlin, 1998.

[18] P. Flandrin. *"Temps-Fréquence". Hermès, 1998.*

[19] I. Daubechies. The wavelet transform, time-frequency localization *and* signal analysis. *IEEE 'Trans. on Inf. Theory,* 36(5):961-1005, 1990.

[20] T. Karp and A. Mertins. "Lifting scheme for biorthogonal modulated filter banks". In *Proc. International Conference on Digital Signal Processing,* Santorini, Greece, July 1997.

[21] T. Karp and A. Mertins. "Efficient filter realizations for cosine modulated filter banks". In *Proc. Colloque GRETSI,* Grenoble, France, September 1997.

CLAIMS

1. A method for transmitting a BFDM/OM biorthogonal multicarrier signal characterized in that it implements a transmultiplexer structure providing:

    -    a modulation step, by means of a bank of synthesis
5   filters (11), having 2M parallel branches, M ≥ 2, each fed by source data and each comprising an expander of order M and filtering means;

    -    a demodulation step, by means of a bank of analysis filters (12), having 2M parallel branches, each
10 comprising a decimator of order M and filtering means, and delivering representative data received from said source data,

    said filtering means being derived from a predetermined prototype modulation function.

15 2. The transmission method according to claim 1, characterized in that said filtering means of said bank of synthesis filters and/or of said bank of analysis filters are grouped as a polyphase matrix, respectively.

3. The transmission method according to claim 2, characterized in that at least one of said polyphase matrices comprises a reverse Fourier transform (51,61) with 2M inputs and 2M outputs.

4. A method for modulating a signal transmitted according to the method of any of claims 1 to 3, characterized in that it implements a reverse Fourier transform (51) fed by 2M source data, each having undergone a predetermined phase shift, and feeding 2M filtering modules, each followed by an expander of order M, the outputs of which are grouped then transmitted.

5. The modulation method according to claim 4, characterized in that it delivers data s[k] such as:

$$x_m^0(n) = a_{m,n} e^{j\frac{\pi}{2}n}$$

$$x_l^1(n) = \sqrt{2} \sum_{k=0}^{2M-1} x_k^0(n) e^{-j\frac{2\pi}{2M}k\frac{D-M}{2}} e^{j\frac{2\pi}{2M}kl}$$

$$= 2M\sqrt{2}\,\mathrm{IFFT}\left(x_0^0(n),\cdots, x_{2M-1}^0(n) e^{-j\frac{2\pi}{2M}(2M-1)\frac{D-M}{2}} \right)$$

$$x_l^2(n) = \sum_{k=0}^{m-1} p(l+2kM) x_k^1(n-2k)$$

$$s[k] = \sum_{n=\left\lfloor\frac{k}{M}\right\rfloor - 1}^{\left\lfloor\frac{k}{M}\right\rfloor} x_{k-nM}^2(n)$$

with α an integer representing the reconstruction delay;

β an integer between 0 and M-1;

and [.] is the "integral part" function.

6. A method for demodulating a signal transmitted according to the method of any of claims 1 to 3 characterized in that it implements a reverse Fourier transform (61) fed by 2M branches, themselves fed by said transmitted signal, and each comprising a decimator of order M followed by a filtering module, and feeding 2M phase shift multipliers, delivering an estimation of the source data.

7. The demodulation method according to claim 6, characterized in that it delivers data $\hat{a}_{m,n-\alpha}$ such that:

$$\hat{x}_l'^2(n-\alpha) = s[nM - \beta - l]$$

$$\hat{x}_l'^1(n-\alpha) = \sum_{k=0}^{m-1} p(l+2kM)\hat{x}_l'^2(n-\alpha-2k)$$

$$\hat{x}_l'^0(n-\alpha) = \sqrt{2}e^{-j\frac{2\pi}{2M}l\frac{D+M}{2}} \sum_{k=0}^{2M-1} \hat{x}_l'^1(n-\alpha)e^{j\frac{2\pi}{2M}kl}$$

$$= 2M\sqrt{2}e^{-j\frac{2\pi}{2M}l\frac{D+M}{2}} \text{IFFT}\left(\hat{x}_l'^1(n-\alpha),\cdots,\hat{x}_{2M-1}'^1(n-\alpha)\right)$$

$$\hat{a}_{m,n-\alpha} = \Re\left\{e^{-j\frac{\pi}{2}(n-\alpha)}\hat{x}_l'^0(n-\alpha)\right\}$$

with: D = 2.s.M + d,

wherein: s is an integer;

d is between 0 and 2M-1.

8. The modulation method according to any of claims 4 and 5, or the demodulation method according to any of claims 6 and 7, characterized in that said filtering

modules are produced as one of the filters belonging to the group comprising:

- transverse structure filters;
- ladder structure filters; and
- trellis structure filters.

9. The modulation method according to any of claims 1 to 8, characterized in that said orthogonal multicarrier signal is a OFDM/OM signal.

10. A device for transmitting and/or receiving a BFDM/OM signal, implementing the method according to any of claims 1 to 9.

(54) Title: METHOD FOR TRANSMITTING AN OFFSET MODULATED BIORTHOGONAL MULTICARRIER SIGNAL (BFDM/OM)

(54) Titre: PROCEDE DE TRANSMISSION D'UN SIGNAL MULTIPORTEUSE BIORTHOGONAL MODULE AVEC OFFSET (BFDM/OM)
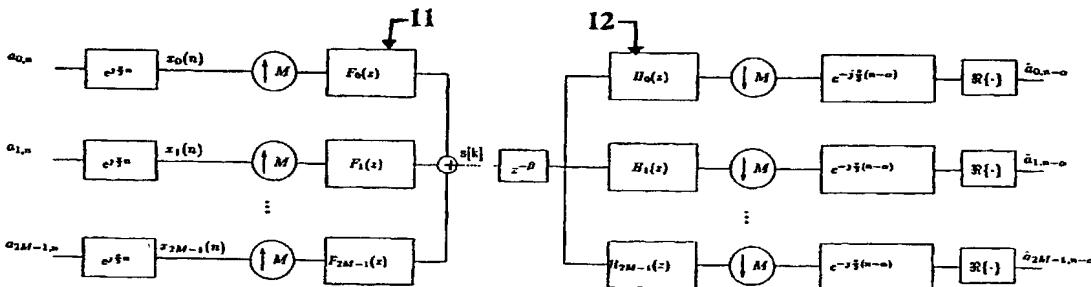
(57) Abstract: The invention concerns a method for transmitting a biorthogonal multicarrier signal BFDM/OM, using a transmul-
tiplexer structure providing: a modulating step, using a synthesis filter bank (11), having two 2M parallel branches, M ≥ 2, each
supplied by source data, and comprising an expander of order M and filtering means; a demodulating step, using an analysis filter
bank (12), having two 2M parallel branches, each comprising a decimation unit of order M and filtering means, and delivering re-
ceived data representing said source data; said filtering means being derived from a predetermined prototype modulating function.

(57) Abrégé: L'invention concerne un procédé de transmission d'un signal multiporteuse biorthogonal BFDM/OM, mettant en
oeuvre une structure de transmultiplexeur assurant: une étape de modulation, à l'aide d'un banc de filtres de synthèse (11), présen-
tant 2M branches parallèles, M≥ 2, alimentées chacune par des données source, et comprenant chacune un expanseur d'ordre M et
des moyens de filtrage; une étape de démodulation, à l'aide d'un banc de filtres d'analyse (12), présentant 2M branches parallèles,
comprenant chacune un décimateur d'ordre M et des moyens de filtrage, et délivrant des données reçues représentatives desdites
données source, lesdits moyens de filtrage étant déduits d'une fonction de modulation prototype prédéterminée.
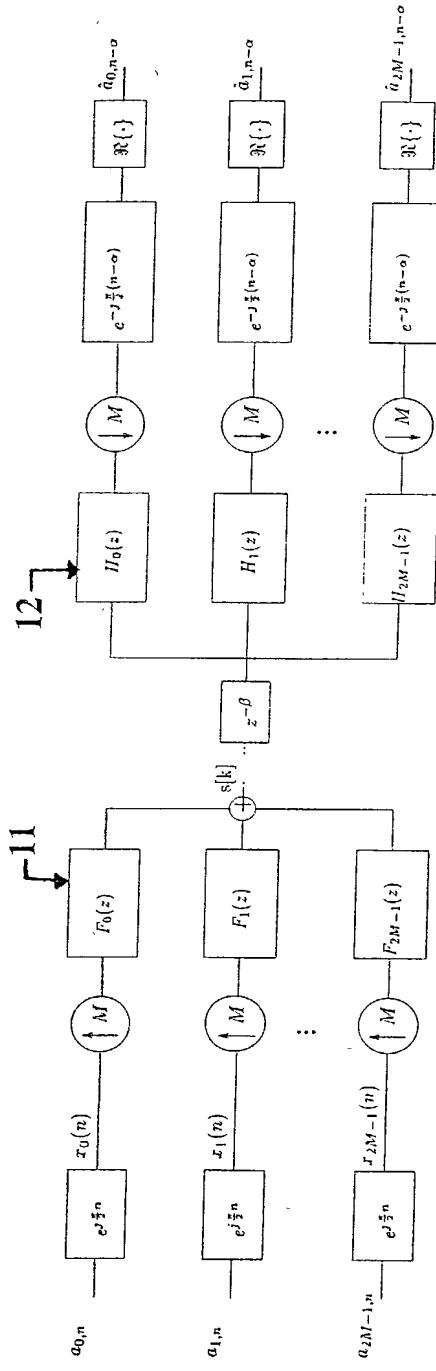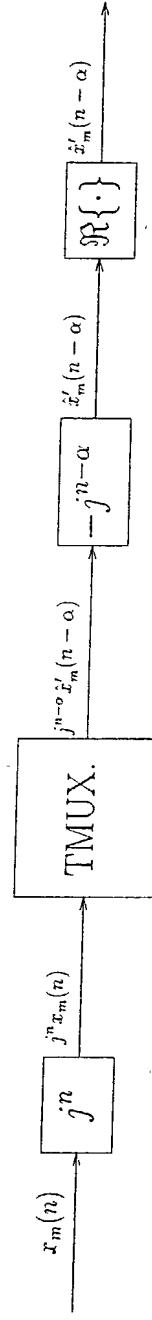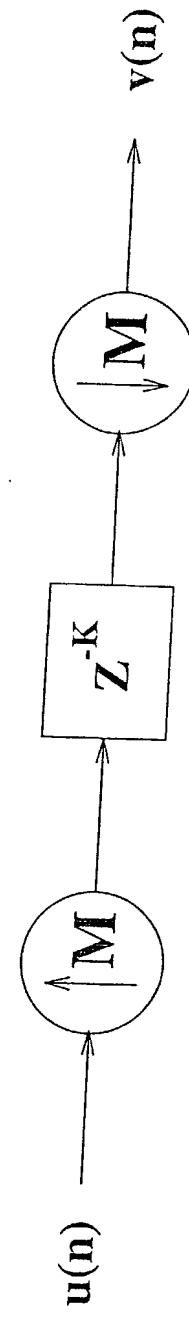
Fig. 1

Fig. 2

Fig. 3

Fig. 4
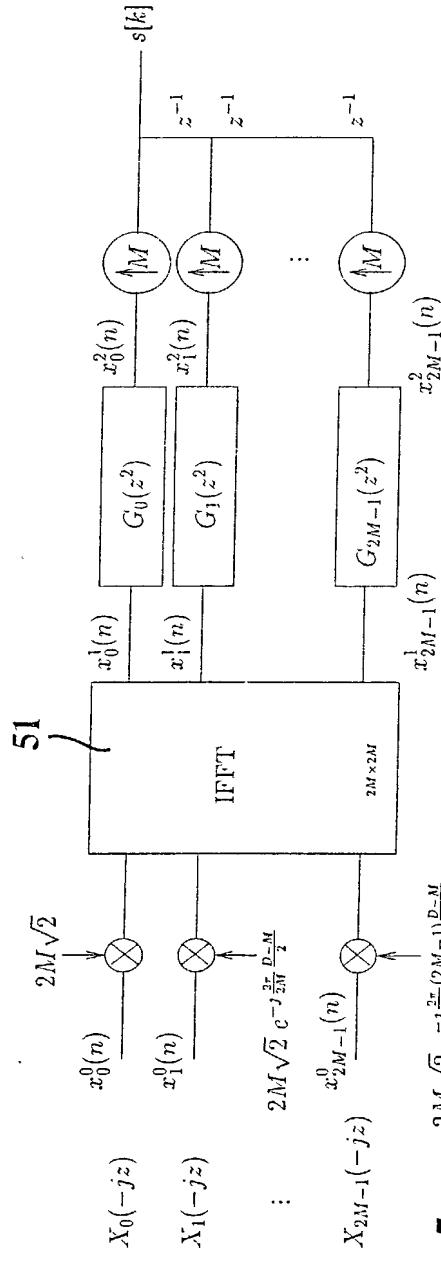
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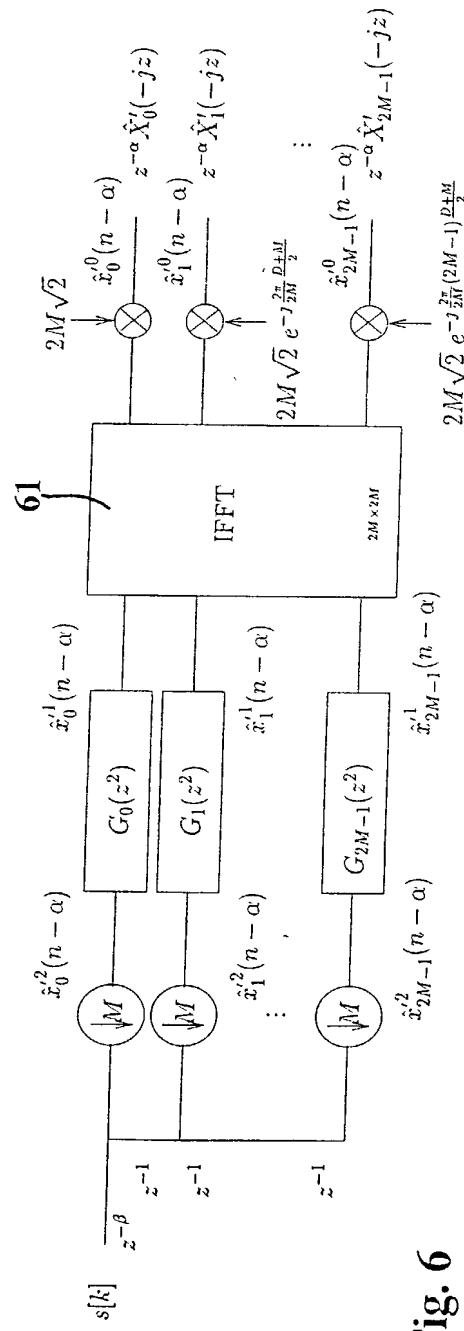
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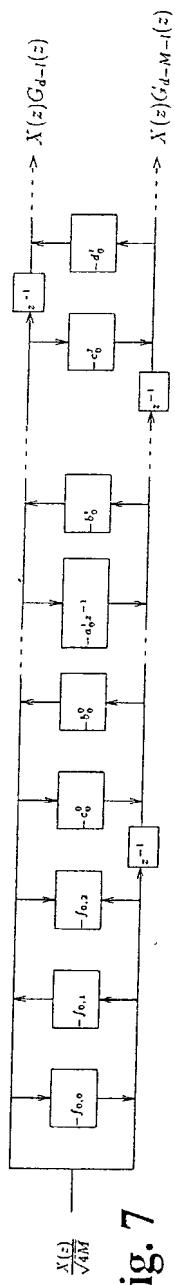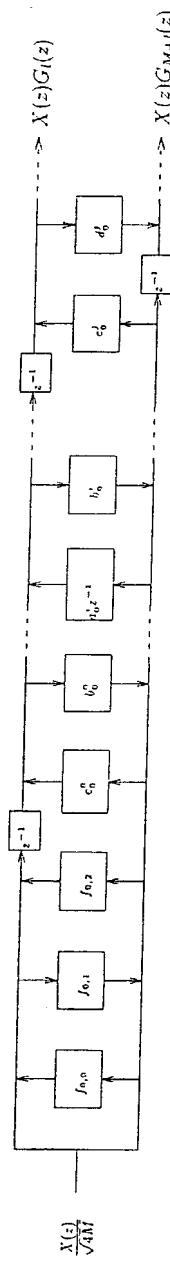
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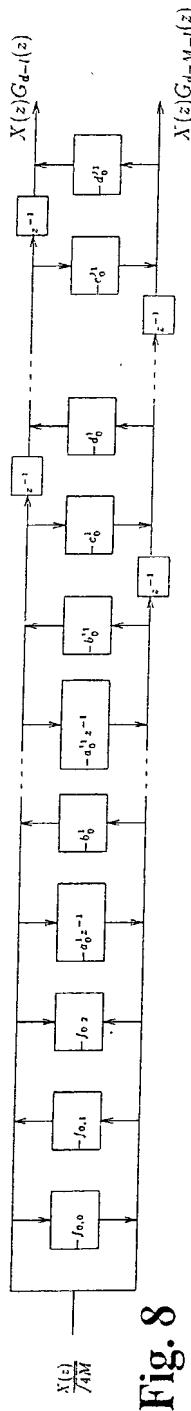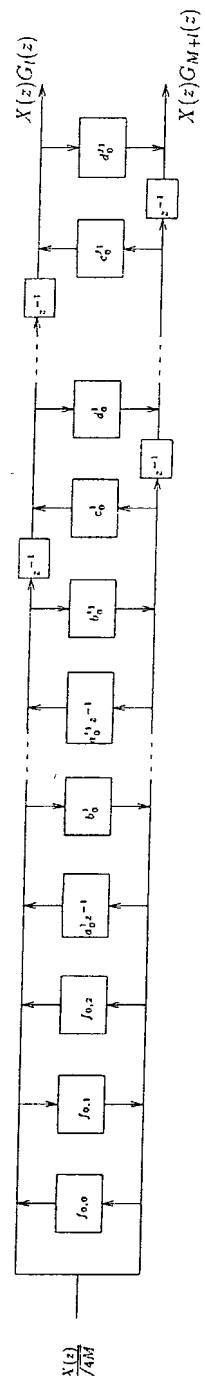
Fig. 8
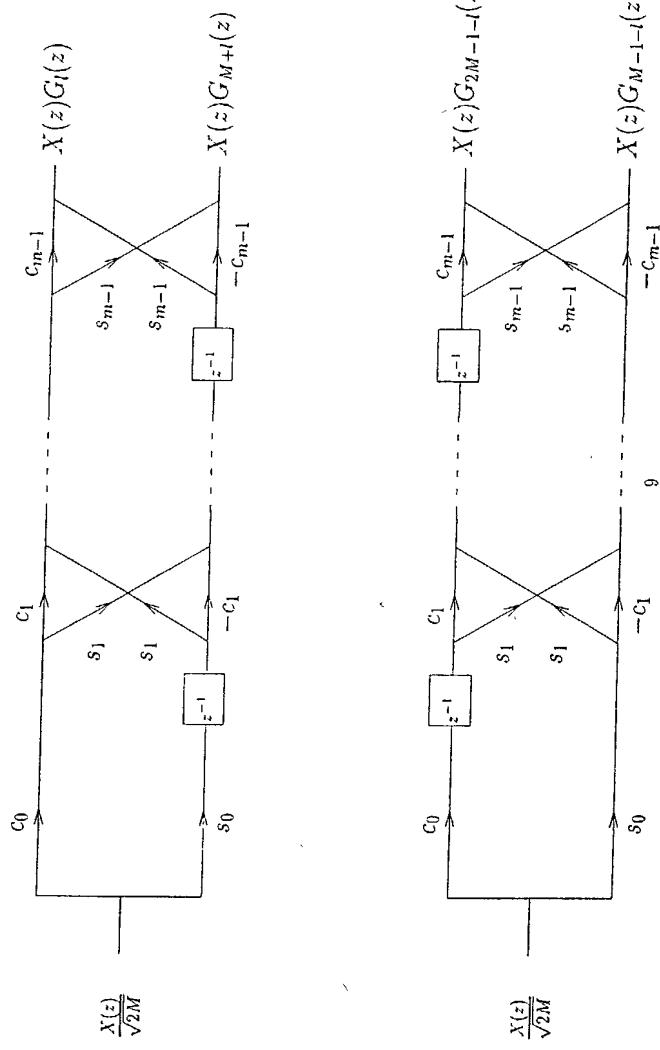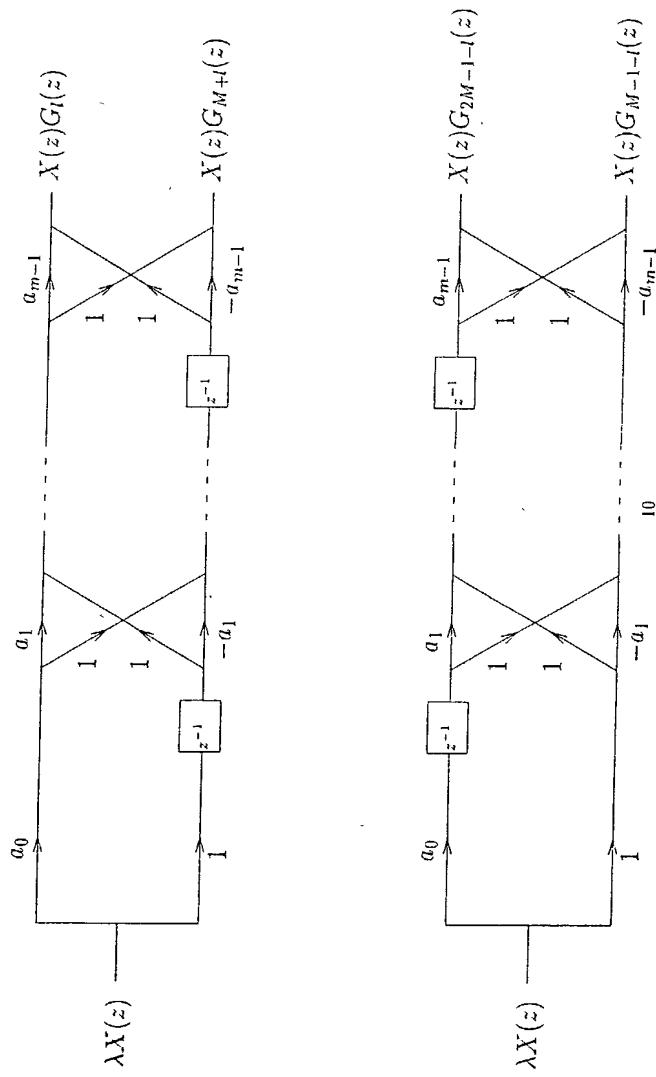
Fig. 9

Fig. 10

Frequency response (module in dB)

Normalized frequency

Fig. 11B

Time response

Discrete time

Fig. 11A

Frequency response (module in dB)

Normalized frequency

Fig. 12B

Time response

Discrete time

**Fig. 12A**

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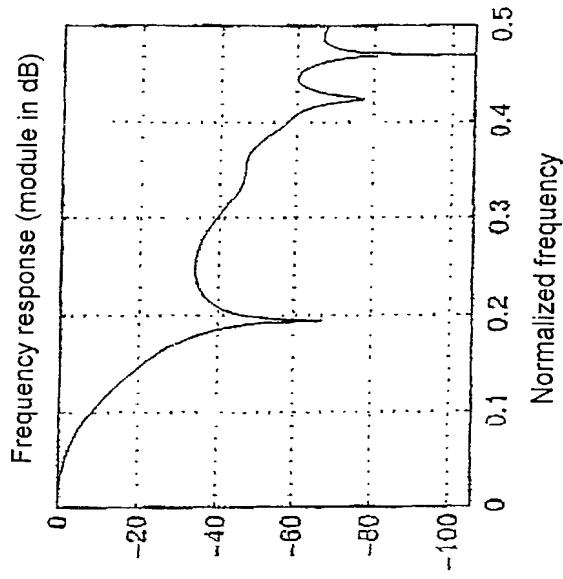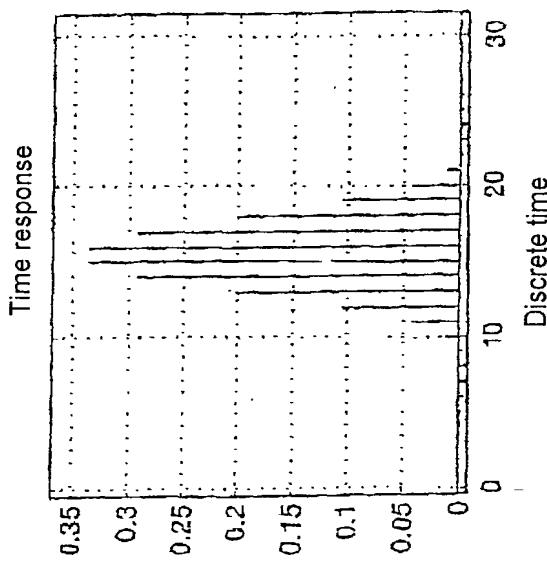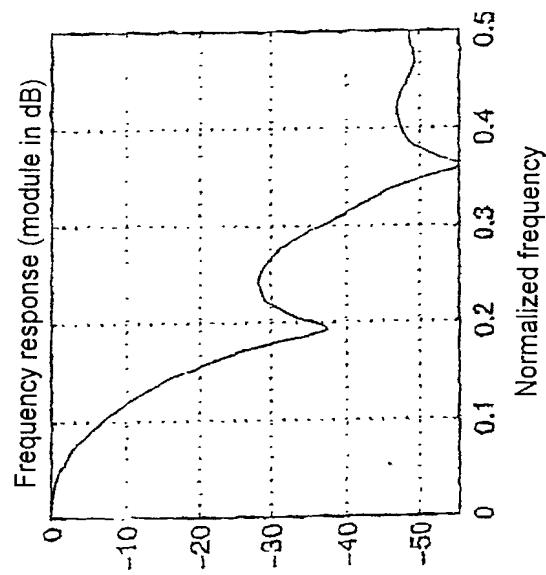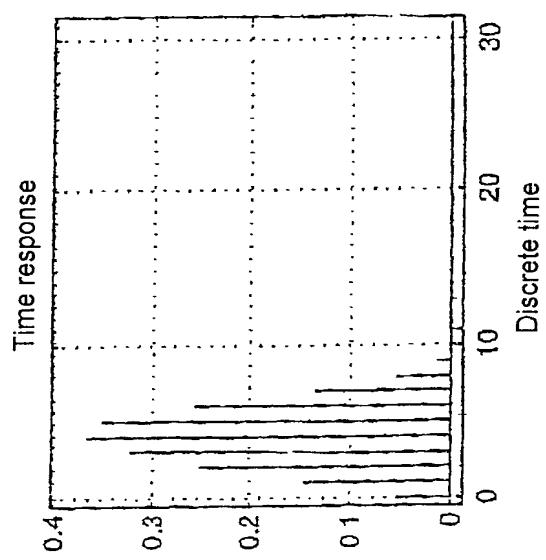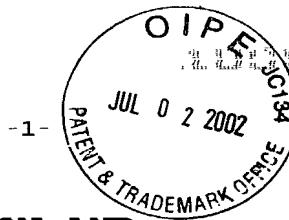
COMBINED DECLARATION AND POWER OF ATTORNEY

IN ORIGINAL APPLICATION

Attorney Docket No.

F40.12-0005

SPECIFICATION AND INVENTORSHIP IDENTIFICATION

As a below named inventor, I declare that:

My residence, post office address and citizenship are as stated below next to my name.

I believe I am the original, first and joint inventor of the subject matter which is claimed, and for which a patent is sought, on the invention entitled __METHOD FOR TRANSMITTING AN OFFSET MODULATED (BFDM/OM) MULTICARRIER SIGNAL__ the specification of which,

(check one) __X__ is attached hereto.

    ____ was filed on _____ as Appln. No. _____.

    ____ and was amended on _____.

    __X__ was described and claimed in PCT International Application No. __PCT/FR00/02716__ filed on __29 September 2000__ and as amended under PCT Article 19 on _____.

ACKNOWLEDGEMENT OF REVIEW OF PAPERS AND DUTY OF CANDOR

I have reviewed and understand the contents of the above identified application, including the claims, as amended by any amendment referred to above. I acknowledge the duty to disclose information which is known to me to be material to the patentability of this application in accordance with 37 C.F.R. § 1.56.

PRIORITY CLAIM (35 U.S.C. § 119)

Prior Foreign Application(s)

I claim foreign priority benefits under 35 U.S.C. § 119(a-d) of any foreign application(s) for patent or inventor's certificate listed below, each of which is incorporated by reference in its entirety, , each of which is incorporated by reference in its entirety, and have also identified below any foreign application for patent or inventor's certificate having a filing date before that of the application on which priority is claimed:

Number	Country	Day/Month/Year Filed	Priority Claimed
99/12371	France	29 September 1999	Yes _X_ No___
			Yes___ No___

Prior Provisional Application(s)

I hereby claim the benefit under 35 U.S.C. §119(e) of any United States Provisional Application(s) listed below, each of which is incorporated by reference in its entirety:

Number   Day/Month/Year Filed

PRIORITY CLAIM (35 U.S.C. § 120)

I claim the benefit under 35 U.S.C. § 120 of any United States application(s) listed below, each of which is incorporated by reference in its entirety. Insofar as the subject matter of each of the claims of this application is not disclosed in the prior United States application in the manner provided by the first paragraph of 35 U.S.C. § 112, I acknowledge the duty to disclose to the Patent Office all information known to me to be material to patentability as defined in 37 C.F.R. § 1.56 which became available between the filing date of the prior application and the national or PCT international filing date of this application:

Appln. No.	U.S. Appl. No. (if any under PCT)	Filing Date	Status
_____	_____	_____	_____
_____	_____	_____	_____

DECLARATION

I declare that all statements made herein that are of my own knowledge are true and that all statements that are made on information and belief are believed to be true; and further that these statements were made with the knowledge that willful false statements and the like so made are punishable by fine or imprisonment, or both, under 18 U.S.C. § 1001 of Title 18 of the United States Code and that such willful false statements may jeopardize the validity of the application or any patent issued thereon.

POWER OF ATTORNEY

I appoint the following attorneys and agents to prosecute the patent application identified above and to transact all business in the Patent and Trademark Office connected therewith, including full power of association, substitution and revocation: Judson K. Champlin, Reg. No. 34,797; Joseph R. Kelly, Reg. No. 34,847; Nickolas E. Westman, Reg. No. 20,147; Steven M. Koehler, Reg. No. 36,188; David D. Brush, Reg. No. 34,557; John D. Veldhuis-Kroeze, Reg. No. 38,354; Deirdre Megley Kvale, Reg. No. 35,612; Theodore M. Magee, Reg. No. 39,758; Christopher R. Christenson, Reg. No. 42,413; Brian D. Kaul 41,885; Robert M. Angus, Reg. No. 24,383; Christopher L. Holt, Reg. No. 45,844; Alan G. Rego, Reg. No. 45,956; and David C. Bohn, Reg. No. 32,015.

I ratify all prior actions taken by Westman, Champlin & Kelly, P.A. or the attorneys and agents mentioned above in connection with the prosecution of the above-mentioned patent application.

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